EXPRESSIONS AND EQUATIONS 3
STUDENT PACKET

EXPRESSIONS, EQUATIONS, AND APPLICATIONS

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

addition property of equality	addition property of zero
additive inverse property	multiplication property of equality
multiplication property of one	multiplicative inverse property

TRAINING FOR A MARATHON

Marathon runners keep track of their progress by measuring "pace" (minutes per mile). T.J. and JoJo are training for an upcoming marathon. They don't usually train together because their paces are so different. JoJo says, "I'll give you a head start. Let's see when I catch up to you." Follow your teacher's directions to complete the page.


JoJo


T.J.

EQUIVALENT EXPRESSIONS: A CUPS AND COUNTERS MODEL

We will apply properties of arithmetic to generate equivalent expressions that include integers and negative coefficients. We will use algebra to investigate two "number tricks."

GETTING STARTED

Match each expression in Column I with an equivalent expression in Column II.

	Column I		Column II
1. ______	$4 + 4$	a.	$x - 1$
2. ______	$4 + (-4)$	b.	$x - (-x)$
3. ______	$x + 1$	c.	$4 - 4$
4. ______	$x + (-1)$	d.	$x - x$
5. ______	$x + x$	e.	$3x - 3x$
6. ______	$x + (-x)$	f.	$4 - (-4)$
7. ______	$3x + 3x$	g.	$x - (-1)$
8. ______	$3x + (-3x)$	h.	$3x - (-3x)$

9. Problems 1-8 illustrate an important relationship between addition and subtraction:

   Subtracting a number gives the same result as …

Simplify each expression.

10. $5x - 2 - 9 + 4x$

11. $-6 + 8(x - 5) - 7x$

CUPS AND COUNTERS: THE UPSIDE-DOWN CUP

Follow your teacher's directions to complete this page.

(1)

Positive Counter	Negative Counter	Cup	Upside-down Cup

(2)	(3)	(4)	(5)
(6)	(7)	(8)	(9)
(10)	(11)	(12)	(13)
(14)		(15)	

x				

16.	17.	18.
19.	20.	21.

PRACTICE 1

Build and draw the following expressions.

1. $-2x + 1$	2. $-x - 4$	3. $-3x + x - 1 + 3$

Write variable expressions for the following.

4. ∧ + +	5. ∧ ∧ ∧ – –	6. ∨ ∧ ∧ – – +

7. Use the distributive property to rewrite each expression as a sum.

	as a product	as a sum		as a product	as a sum
a.	$-2(x + 6)$		d.	$-4(2x - 1)$	
b.	$-5(x - 1)$		e.	$-(-x + 1)$	
c.	$7(-x + 2)$		f.	$-(4 - 3x)$	

8. Consider the algebraic expression: $x + 3x - 2(x + 4) - 1$

   a. Evaluate this expression when $x = 10$.

   b. Simplify the algebraic expression by combining like terms.

   c. Evaluate the simplified expression in part b when $x = 10$.

   d. Did you end up with the same value in parts a and c? ______ Why?

Simplify each expression.

9. $-(x - 1) + 5(x - 1)$	10. $8 + 8x - 6(x + 2)$	11. $4(2x - 1) + 3x - 5$

NUMBER TRICKS

Follow your teacher's directions to complete this page.

Step	Abbreviate action	Arithmetic	
a.			
b.			
c.			
d.			
e.			
f.			
g.			

What's your final result?

What did others in class get?

What's the "trick"?

Step	Abbreviate action	Arithmetic	
a.			
b.			
c.			
d.			
e.			
f.			
g.			

What's your final result?

What did others in class get?

What's the "trick"?

SOLVING EQUATIONS: FROM BALANCE TO PROPERTIES

We will simplify expressions within equations. We will solve equations that include negative coefficients using balance techniques. We will formalize balance techniques with properties of equality.

GETTING STARTED

1. Explain what it means to substitute the value $p = -2$ into the expression $3p + 9$.

2. Explain how you know that $m = 6$ is the solution to the equation $7m = 42$.

3. Explain how you know that $n = 4$ is the solution to this equation $2n + 6 = 14$.

4. Solve the equation $-44 = 5(w + 2) + w$ using any strategy that you have learned.

CAN YOU SOLVE THIS EQUATION?

Solve this equation using any strategy.

If you want to use an organized guess-and-check strategy, use the table below.

$$-4(x - 2) + 6 = 20 - 2(x - 1) - 7$$

Value for x	Value of expression for **left** side of equation	Value of expression for **right** side of equation	Difference between left side and right side	Are both sides equal?

EQUATION SOLVING STRATEGY:
FROM BALANCE TO PROPERTIES

Follow your teacher's directions to solve these problems.

Equation	Actions/Properties Used
(1)	
(2)	
(3)	
(4)	

(5) Look up and record the properties discussed in these problems in My Word Bank.

EQUATION SOLVING STRATEGY: FORMAL ALGEBRA

Use formal algebra to solve each equation and check it. Build or draw as needed.

Equation	Picture (if needed) and Check
(1) $6x - 4 = 2x - 8$	Check:
2. $2(x - 3) = 4(x + 1)$	Check:
3. $-(x - 6) = 3x - 2$	Check:
4. $-4(x - 1) = -3(x + 1)$	Check:

PRACTICE 2

Use formal algebra to solve each equation. Check by substitution. Build or draw as needed.

1. $x - 4 = 2x + 1$	2. $2(x - 1) = 2x + 1 + 3x$
3. $-3x - 6 = 3 - 6x$	4. $-(x - 1) = -2x - 5$
5. $3(x + 1) = -5(x + 1)$	6. $-x - 4 - x = -4(x - 1)$

PRACTICE 3

Solve each equation algebraically. Check each solution.

1. $200 + 30x = 100 + 50x$	2. $5(x + 20) = -10x + 70$
3. $10(2x – 1) = 5(6x + 8)$	4. $-2(10x + 4) = -4(6x – 8)$

5. Two friends, Ahn and Felipe, are both saving for the same phone. On their graduation day, Ahn received a \$200 gift and Felipe received a \$100 gift. Ahn will save \$30 per week and Felipe will save \$50 per week.

   a. Fill in the table below to show how much money each has after saving for 8 weeks.

	0								
	200								
	100								

   b. At what week do they have the same amount of money? ______

   c. Explain how this problem relates to the equation in problem 1 above.

APPLICATIONS

We will write equations to solve problems. We will apply what we've learned about formal algebra to solve equations that involve rational numbers.

GETTING STARTED

Two friends, Sadie and Betina, are both saving for the same phone. On their graduation day, Sadie received a $200 gift and Betina received a $150 gift. Sadie will save $40 per week and Betina will save $25 per week.

1. Fill in 8 weeks of entries for the problem above.

weeks →	0	1	2	3	4	5	6	7	8
Sadie									
Betina									

2. Who started with more money?

3. Write an expression for the amount of money Sadie will save after *w* weeks.

4. Write an expression for the amount of money Betina will save after *w* weeks.

5. Explain why Betina's savings will never catch up to Sadie's?

MORE SAVINGS PROBLEMS

Use the following organizational structure to solve these problems with algebra.

- Identify the variable(s).
- Write an equation.
- Solve the equation.
- Answer the question.
- Check your answer in the ORIGINAL problem.

1. Sven is saving for a \$608 phone (tax included). He has \$118 already and will save \$35 per week. After how many weeks of saving will he be able to purchase the phone?

2. Ben is saving for the same phone and wants to buy it at the same time as Sven. Ben already has \$73 and will save the same amount week. How much should his weekly savings be?

3. Because the Philadelphia Eagles won Super Bowl LII, their ticket prices are going up for the 2018-19 season. Barry and Ryann are eager to buy some. Barry has \$400 saved already and starts to save another \$20 per week. At the same time Ryann starts to save \$40 per week, but she has a \$100 debt to pay off to her brother first before the savings are truly hers to keep. After how many weeks of saving does Ryann catch up to Barry?

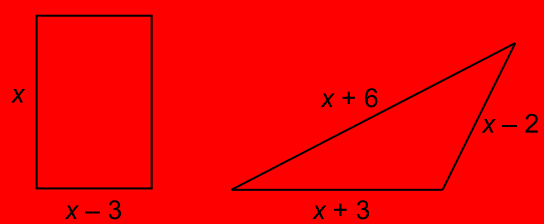
4. Christina and Dylan want to buy Eagles tickets too, but they both owe their friend Cocoa money before they can start saving. Christina owes \$50 and saves \$60 per week. Dylan owes \$75 and saves \$65 per week. After how many weeks do they have the same amount of money? And how much money will they each have at that time?

PERIMETER PROBLEMS

Use the same organizational structure as the previous page to solve the following problems.

- Identify the variable(s).
- Write an equation.
- Solve the equation.
- Answer the question.
- Check your answer in the ORIGINAL problem.

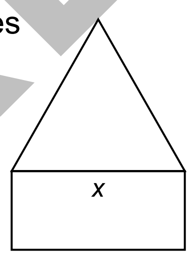
1. This rectangle and this triangle have the same perimeter. What is the perimeter of each?



2. In a rectangle, the base is 5 units more than the height, *x*. Twice the height is equal to the base. What are the dimensions of the rectangle and its perimeter?

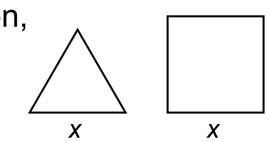
3. An equilateral triangle sits on top of a rectangle to form a “house.” The height of the rectangle is 6 units less than its base. The perimeters of both figures are equal.

   What is the perimeter of the “house”?



4. Suppose the square and the equilateral triangle below have the same side lengths and the same perimeters. Write an equation that describes this statement.

   Then solve the equation, and use the solution to explain why the original statement cannot be true.



TRAINING FOR A MARATHON REVISITED

Look back to the opening problem, Training for a Marathon.

1. T.J. is an average runner whose pace is _____ minutes per mile.
2. JoJo is a wheelchair athlete whose pace is _____ minutes per mile.
3. T.J. gets a _____ minute head start from JoJo.

Another friend, Bryce, wants to train for the marathon too. He is a walker, whose pace is 15 minutes per mile.

4. List the athletes from fastest to slowest. ___________, ___________, ___________
5. How long does it take each to go 6 miles?

   T.J. _____ min; JoJo _____ min; Bryce _____ min;

6. How long does it take each to go x miles?

   T.J. _____ min; JoJo _____ min; Bryce _____ min;

Solve the following problems.

7. On another training session, JoJo says to Bryce, "I'll give you a 30 minute head start." Let x = # of miles. Write and solve an equation to determine when JoJo catches up to Bryce. Explain the solution in the context of the problem and how much time each has trained.	8. The following week, T.J. says to Bryce, "I'll give you a 40 minute head start." Let x = # of miles. Write and solve an equation to determine when T.J. catches up to Bryce. Explain the solution in the context of the problem and how much time each has trained.

EQUATIONS INVOLVING RATIONAL NUMBERS

Use your knowledge of fractions and decimals to help you solve these equations using formal algebra. Check each solution by using substitution.

1. $-x - 1 = x - 2$	2. $1 + x = 3(x - 2) - 2$
3. $-6(-2x + 1) = -4(7x - 1)$	4. $-4(x - 2) + 2 = -2(x - 1) - 5$
5. $5x + 0.4 = 2x + 0.7$	6. $0.2(x + 0.3) = 0.4(x - 0.1)$
7. $6x - \frac{1}{8} = 4x + \frac{1}{4}$	8. $\frac{1}{2}(2x + 4) + x = \frac{1}{3}(9x + 12)$

PRACTICE 4

Solve using formal algebra. Check each solution by using substitution.

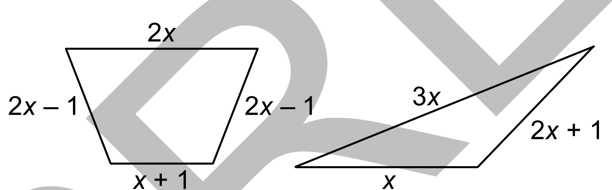
1. $350x - 150 = -450x + 250$

2. $-6x - 16 = 12(x - 2)$

3. $-0.5(x - 6) = 1.5x - 1.2$

4. $-\frac{1}{2}(x + 8) = \frac{1}{4}(x - 4)$

5. These figures have the same perimeter. What is the perimeter of each?



6. Keiko has \$400 and is saving \$40 per week. Dev has \$300 and starts saving at the same time as Keiko, saving \$30 per week. Write an equation that would show when they have saved the same amount. Then solve the equation, and explain why this solution does not make sense.

REVIEW

EXPRESSION GAME

Play five rounds to see who gets the most wins. Record each round in the table below.

1. **Player 1** rolls a number cube for an expression below.

If →	⚀	⚁	⚂	⚃	⚄	⚅
then →	$-2x - 4$	$-2(x - 4)$	$4x + 2$	$4(x + 2)$	$-3(x + 1)$	$-3x + 1$

2. **Player 2** rolls for an expression below.

If →	⚀	⚁	⚂	⚃	⚄	⚅
then →	$-x - 3$	$-(x - 3)$	$5(x - 1)$	$-5(x - 1)$	$-6x + 4$	$6(-x + 4)$

3. **Both** players add these two expressions to get a sum. Use extra paper if needed. Check that you both agree!

4. **Both** players roll for their own *x*-value below.

If →	⚀	⚁	⚂	⚃	⚄	⚅
then →	$x = 1$	$x = 2$	$x = 3$	$x = -1$	$x = -2$	$x = -3$

5. Players substitute their own *x*-value into the expression sum and evaluate. Use your own paper if needed.

6. The player with the greater value in step 5 wins the round.

		Round 1	Round 2	Round 3	Round 4	Round 5
1.	**Expression Player 1**					
2.	**Expression Player 2**					
3.	**Expression Sum**					
4.	**My *x*-value**					
5.	**Substitute and evaluate**					
6.	**Winner**					

POSTER PROBLEM: SOLVING EQUATIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)
$-4(x + 3) = -2(x - 6)$	$-6(x + 5) = -2x - 10 - 8x$
Poster 3 (or 7)	Poster 4 (or 8)
$5(-x + 4) = -5(4 - x)$	$-10(3x + 1) = -20(2x - 1)$

Copy the problem, then:

A. Simplify the expressions on both sides of the equation using properties of arithmetic.

B. Perform one step to both sides of the equation using properties of equality.

C. Perform another step to both sides of the equation using properties of equality.

D. Find the solution to the equation.

Part 3: Return to your seats. Work with your group, and show all work.

Use your "start problem." Check each step to double check that everything was done correctly. Then substitute the solution back into the original equation to verify that it is correct.

MULTI-STEP TARGET EQUATIONS

For problems 1-2, use this equation structure: $\square x + \square = \square x + \square$

1. Using exactly four of the digits 1 through 9 one time each, write two different equations with positive solutions.

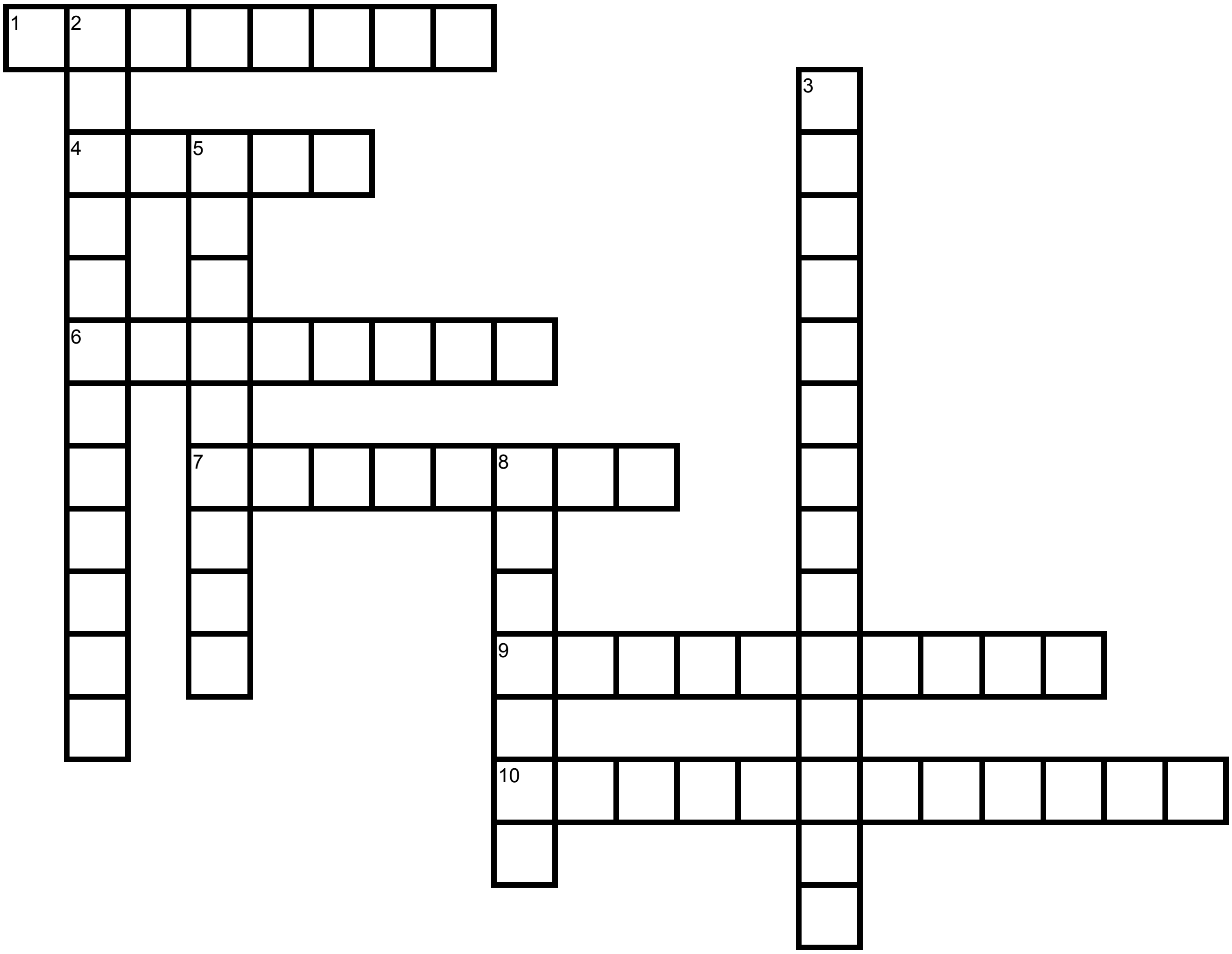
2. Using exactly four of the digits 1 through 9 one time each, write two different equations with negative solutions.

For problems 3-4, use this equation structure: $\frac{\square}{\square}\left(\square x + \square\right) = \square x + \square$

3. Using exactly six of the digits 1 through 9 one time each, write two different equations with positive solutions.

4. Using exactly six of the digits 1 through 9 one time each, write two different equations with negative solutions.

VOCABULARY REVIEW



Across

1 The ___ property of equality tells us that we can add the same quantity to, or subtract the same quantity from, both sides of an equation.)

4 To find a value that makes an equation true.

6 The multiplication property of one is also called the multiplicative ____ property.

7 A statement asserting that two expressions are equal.

9 Since $3(x + 2)$ and $3x + 6$ are equal for all values of x, we say that these expressions are ____.

10 ____ refers to replacing a quantity with one that is equal to it.

Down

2 $3(x + 2) = 3x + 6$ is an example of the ___ property.

3 The ___ property of equality tells us that we can multiply or divide both sides of an equation by the same quantity.

5 The expressions $5x$ and $7x$ have the same variable part, so we say that they are ___ ___ (2 words). This is NOT true for the expressions 9 and $6x$.

8 The additive ___ property tells us that the sum of a number and its opposite are zero.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
addition property of equality	The addition property of equality states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. If $4 + 3 = 7$ and $10 = 2 \bullet 5$ then $(4 + 3) + 10 = 7 + 2 \bullet 5$ check $17 = 17$
addition property of zero	The addition property of zero states that $a + 0 = 0 + a = a$ for any number a. In other words, the sum of a number and 0 is the number. We say that 0 is an additive identity. The addition property of zero is sometimes called the additive identity property. $3 + 0 = 3$ $0 + 7 = 7$ $-5 + 0 = -5 = 0 + (-5)$ $5 + (-5) + 7 = 0 + 7 = 7$
additive inverse property	The additive inverse property states that $a + (-a) = 0$ for any number a. In other words, the sum of a number and its opposite is 0. The number $-a$ is the additive inverse of a. $3 + (-3) = 0$ $-25 + 25 = 0$ $2x + (-2x) = 0$
multiplication property of equality	The multiplication property of equality states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. If $4 + 3 = 7$ and $10 = 2 \bullet 5$ then $(4 + 3) \bullet 10 = 7 \bullet 2 \bullet 5$ check $70 = 70$
multiplication property of 1	The multiplication property of 1 states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. The multiplication property of 1 is sometimes called the multiplicative identity property. $4 \bullet 1 = 4$ $1 \bullet (-5) = -5$
multiplicative inverse	For $b \neq 0$, the multiplicative inverse of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \frac{1}{b} = 1$. The multiplicative inverse of b is also referred to as the reciprocal of b. The multiplicative inverse of 4 is $\frac{1}{4}$, since $4 \bullet \frac{1}{4} = 1$.
multiplicative inverse property	The multiplicative inverse property states that $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$ for every number $a \neq 0$. See multiplicative inverse. $25 \bullet \frac{1}{25} = \frac{1}{25} \bullet 25 = 1$

The Cups and Counters Model for Expressions and Equations

- This cup $\boxed{\vee}$ represents an unknown (or x).

  Draw the cup like this: $\vee$

- This upside-down cup $\boxed{\wedge}$ represents the opposite of the unknown (or $-x$).

  Draw the upside-down cup like this: $\wedge$

- This counter $\boxed{+}$ represents 1 unit (or 1).

  Draw the counter like this: +

- This counter $\boxed{-}$ represents the opposite of 1 unit (or -1).

  Draw the counter like this: –

Zero pairs illustrate the additive <u>inverse property</u>.

Here are some examples:

counters	counter drawing	cups	cups drawing
$\boxed{-}\boxed{+}$ $\boxed{+}\boxed{-}$	– + + –	$\boxed{\wedge}\boxed{\vee}$ $\boxed{\vee}\boxed{\wedge}$	$\wedge$ $\vee$ $\vee$ $\wedge$
$2 + (-2) = 0$	$2 + (-2) = 0$	$2x + (-2x) = 0$	$2x + (-2x) = 0$

Attend to Precision When Using the Words Negative and Opposite

Saying "negative x" improperly implies that $-x$ represents a negative number.

Consider:

- If $x = 4$, then $-x = -4$.
- If $x = -5$, then $-x = -(-5) = 5$.
- If $x = 0$, then $-x = -0 = 0$.

In only the first example above is $-x$ a negative number.
Instead, it is appropriate to say "the opposite of x."
We may also say "minus x," which is short for minus sign followed by x.

Simplifying Expressions Using a Model

In mathematics, to simplify a numerical or algebraic expression is to convert the expression to a less complicated form.

We can illustrate simplifying expressions using a cups and counter model.

Example 1:

Simplify: $3(x + 2) + 1$

Picture	Expression	What did you do?
V + + **V + +** **V + +** **+**	$3(x + 2) + 1$ $= 3x + 6 + 1$ $= 3x + 7$	Build the expression (3 groups of $x + 2$ and then one more) Simplify (Use the distributive property and add like terms.)

Example 2:

Simplify: $-(x – 3)$

Picture	Expression	What did you do?
V – – – ⇩ **Λ + + +**	$x – 3$ ⇩ $- (x – 3)$ $= -x + 3$	First build the expression inside the parentheses. Then rebuild its opposite to remove parentheses. (the distributive property) OR apply the distributive property immediately. Since $-(x – 3) = -x + 3$, simply build an upside-down cup and 3 positive counters.

Example 3:

Simplify: $2x – 3(x – 4)$

Picture	Expression	What did you do?
V V Λ + + + + **Λ + + + +** **Λ + + + +**	$2x – 3(x – 4)$ $= 2x – 3x + 12$ $= -x + 12$	Build the expression (think: $2x$ and the opposite of 3 groups of $x – 4$). Remove zero pairs. OR apply the distributive property (no action is taken with the model here); then collect like terms to simplify.

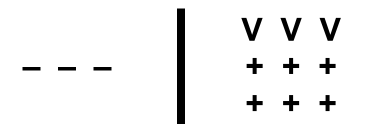
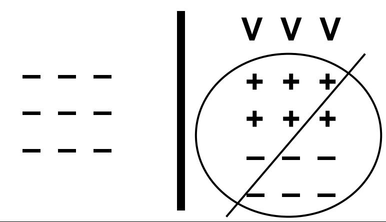
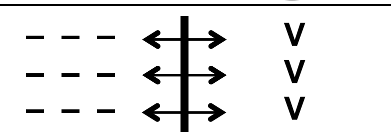
Solving Equations Using a Model

Let **+** represent 1 Let **V** represent the unknown

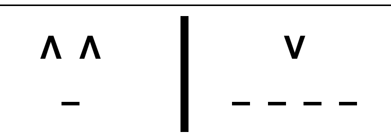
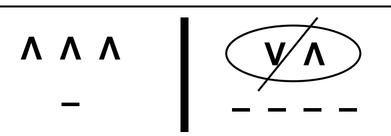
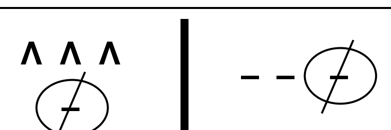
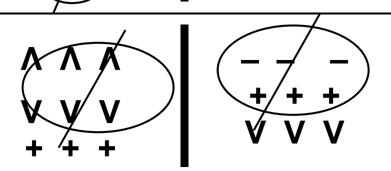
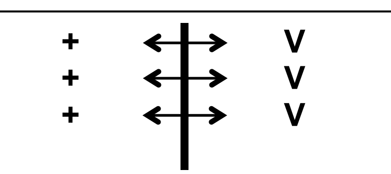
Let **–** represent -1 Let **Λ** represent the opposite of the unknown

The following examples illustrate one solution path. Others paths are possible to arrive at the same solutions.

Example 1:
Solve: $-3 = 3x + 6$
Check (after solving): $-3 = 3(-3) + 6 = -9 + 6 = -3$

Picture	Equation	What did you do?
	$-3 = 3x + 6$	Build the equation.
	$-3 = 3x + 6$ $\underline{+(-6)} = \underline{+(-6)}$ $-9 = 3x$	Add -6 to each side. Remove zero pairs.
	$\frac{-9}{3} = \frac{3x}{x}$ $-3 = x$	Divide both sides by 3. Put counters equally into cups.

Example 2:
Solve: $-2x - 1 = x - 4$
Check (after solving): $-2(1) - 1 = 1 - 4 \rightarrow -2 - 1 = -3 \rightarrow -3 = -3$

Picture	Equation	What did you do?
	$-2x - 1 = x - 4$	Build the equation.
	$-2x - 1 = x - 4$ $\underline{+(-x)} = \underline{+(-x)}$ $-3x - 1 = -4$	Add the opposite of *x* to both sides. Remove zero pairs.
	$-3x - 1 = -4$ $\underline{-(-1)} = \underline{-(-1)}$ $-3x = -3$	Remove -1 from both sides. (This gives the same result as adding 1 to each side.)
	$-3x = -3$ $\underline{(+3x) + 3} = \underline{+3 + (3x)}$ $3 = 3x$	Add the opposite of x to both sides AND add 3 positives to both sides
	$\frac{3}{3} = \frac{3x}{3}$ $1 = x$	Divide both sides by 3. Put counters equally into cups.

Summary of Properties Used for Solving Equations	
Properties of arithmetic govern the manipulation of expressions (mathematical phrases). These include:	
• Associative property of addition	• Associative property of multiplication
• Commutative property of addition	• Commutative property of multiplication
• Additive identity property	• Multiplicative identity property
• Additive inverse property	• Multiplicative inverse property
• Distributive property relating addition and multiplication	
Properties of equality govern the manipulation of equations (mathematical sentences). These include:	
• Addition property of equality (Subtraction property of equality)	• Multiplication property of equality (Division property of equality)

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g., associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g., addition property of equality, multiplication property of equality).

Solve: $3 - x + 3 = 5x - 2x - 2$ for x

Equation	Comments
$3 - x + 3 = 5x - 2x - 2$ $6 - x = 3x - 2$	• Collect like terms ($3 + 3 = 6$; $5x - 2x = 3x$). Note that this is an application of the distributive property because $(5 - 2)x = 3(x)$.
$6 - x = 3x - 2$ $\underline{+2} = \underline{+2}$ $8 - x = 3x$	• Addition property of equality (add 2 to both sides) • Additive inverse property ($-2 + 2 = 0$)
$8 - x = 3x$ $\underline{+x} = \underline{+x}$ $8 = 4x$	• Addition property of equality (add x to both sides) • Additive inverse property ($-x + x = 0$) • Collect like terms ($3x + x = (3 + 1)x = 4x$).
$8 = 4x$ $\frac{8}{4} = \frac{4x}{4}$ $2 = x$	• Multiplication property of equality (divide both sides by 4 or multiply both sides by $\frac{1}{4}$) • Multiplicative identity property ($1x = x$)

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